The consequences of increases in the scale of tax and transfer programs are assessed in the context of a model with idiosyncratic productivity shocks and incomplete markets. The effects are contrasted with those obtained in a stand-in household model featuring no idiosyncratic shocks and complete markets. The main finding is that the impact on hours remains very large, but the welfare consequences are very different. The analysis also suggests that tax and transfer policies have large effects on average labor productivity via selection effects on employment.

1. Introduction

Following Prescott (2004), a recent literature has assessed the extent to which cross-country differences in the size of tax and transfer programs can account for the large differences in hours of work across countries. Much of this work uses the neoclassical growth model with a stand-in household. Given its status as the benchmark model of modern macroeconomics, it is a natural starting point for evaluating the aggregate effects of tax and transfer programs. However, it is also of interest to analyze the extent to which deviations from this model might influence our assessment of the effects on aggregate allocations and welfare. The goal of this paper is to carry out such an analysis in another framework that has become very popular for addressing macroeconomic issues: the heterogeneous agents/incomplete markets model of Huggett (1993) and Aiyagari (1994), extended to allow for an endogenous labor supply decision.

The key feature of this model is that individuals face idiosyncratic shocks to their productivity in the market sector, but do not have access to insurance markets. To smooth consumption in the face of these productivity shocks, individuals can vary their saving behavior and their labor supply behavior. Previous work using this model suggests that it may have interesting implications for the analysis of tax and transfer programs. First, from the perspective of how tax and transfer programs affect allocations, Chang and Kim (2006, 2007) find that the heterogeneity induced by idiosyncratic productivity shocks plays an important role in determining the aggregate labor supply elasticity. Second, from the perspective of how tax and transfer programs affect welfare, Pijoan-Mas (2006) shows that in the absence of taxes, the competitive equilibrium in this model has inefficiently high labor supply, due to the fact that individuals rely at least partly on labor supply to smooth consumption in the face of negative productivity shocks.

We calibrate the model and analyze the effects of a simple tax and transfer scheme that places a proportional tax on labor earnings that is used to fund a lump sum transfer that is uniform across all individuals. The main findings are as
follows. First, from the perspective of accounting for differences in hours worked between the US and European countries such as Belgium, France, Germany and Italy, the results are somewhat stronger than in the typical stand-in household model. Specifically, an increase in the labor tax rate from 0.30 to 0.50 leads to a drop in hours worked of 27%, versus 21% in the stand-in household model. Second, the welfare implications are very different. Whereas the stand-in household model implies that optimal allocations are achieved with zero taxes and transfers, the heterogeneous agents/incomplete markets model implies that a substantial tax and transfer program has the ability to enhance steady state welfare. While moving from an American sized tax and transfer scheme to a European sized tax and transfer scheme still implies a welfare loss, the magnitude of the loss is smaller by a factor of three. Third, tax and transfer programs are found to have a substantial positive impact on output per hour, due to the selection effects associated with the effects of tax and transfer schemes on employment. In light of this result, it may be that the apparent catchup of several European countries to the US in terms of productivity may be an illusion.

Our analysis is similar in spirit to Ljungqvist and Sargent (2006, 2008), who also investigate the extent to which uncertainty and incomplete markets affect the findings of earlier studies. The key difference between their analysis and ours is in the type of uncertainty that is considered. Whereas Ljungqvist and Sargent consider stochastic transitions between two levels of skill in a life cycle model, we calibrate our model to match all movements in productivity. Similar to them, we find that uncertainty and incomplete markets have relatively little effect on the magnitude of the response of aggregate employment, but that there are interesting implications for the identity of the individuals who do change their hours of work. They did not carry out a welfare analysis. Our paper is perhaps most similar to Floden and Linde (2001), who also assess the impact of changes in tax and transfer programs in a model with idiosyncratic shocks and incomplete markets. Our model differs from theirs in that we do not allow for permanent heterogeneity and we assume that labor is indivisible. We also focus on some additional aspects, including, for example, the implications for labor productivity.

An outline of the paper follows. The next section summarizes the effects of changes in tax and transfer programs in the stand-in household model. Section 3 describes the incomplete markets model with idiosyncratic shocks. Section 4 calibrates the model and reports the results of how changes in tax and transfer programs influences aggregate allocations and welfare. Section 5 focuses on the implications for productivity, and Section 6 concludes.

2. Taxes and transfers in the stand-in household model

For purposes of comparison this section briefly considers the benchmark model. A stand-in household has preferences

$$\sum_{t=0}^{\infty} \beta^t [\log(c_t) + z\log(1-h_t)]$$

Technology is described by an aggregate production function, $y_t = k_t^\theta h_t^{1-\theta}$. Output can be used as either consumption or investment, and capital depreciates at rate $\delta$.

Following Prescott (2004) it is assumed that a government taxes labor earnings at the constant proportional rate $\tau$ and uses the proceeds to finance a lump-sum transfer $T$ using a period-by-period balanced budget rule, $T_t = \tau w_t h_t$, where $w_t$ is the period $t$ wage.

The analysis focuses on the steady state equilibrium of this economy. One can easily show that increases in $\tau$ have no impact on $k/h$ but do lead to lower $h$. To assess the quantitative implications, consider a standard calibration of the above model to the US economy. Based on McDaniel (2006), let $\tau = 0.30$ as a typical value for the effective labor tax rate in the US. Letting a period denote a year, choose values for $\beta, \theta, \delta$, and $z$ so as to match targets for the real rate of return to capital (4%), capital's share of income (0.36), the share of consumption in output (0.75), and fraction of time devoted to market work (0.33) in the steady state. The implied parameter values are $\beta = 0.96, \theta = 0.36, \delta = 0.096$ and $z = 1.21$. The implied capital/output ratio is 2.62, which is well within the range of standard estimates.

Steady state welfare consequences of different tax rates are expressed as the proportional increase in consumption, denoted by $A$, required to leave the household indifferent between the two steady state allocations. A positive value of $A$ indicates that welfare is lower than in the $\tau = 0.3$ steady state, while a negative value for $A$ indicates that welfare is higher than in the $\tau = 0.3$ steady state. Table 1 presents results.

For future reference, note that since changes in $\tau$ do not have any effect on $k/h$, they also do not impact $k/y$ or $y/h$, so these statistics are not reported. The effect of taxes on hours worked is large: starting from the $\tau = 0.3$ steady state, the effect of a 10 percentage point change in tax rates is roughly a 10% decrease in aggregate hours. In this model the

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$h(\tau)/h(0.3)$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.17</td>
<td>-0.06</td>
</tr>
<tr>
<td>0.2</td>
<td>1.09</td>
<td>-0.03</td>
</tr>
<tr>
<td>0.3</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.4</td>
<td>0.90</td>
<td>0.05</td>
</tr>
<tr>
<td>0.5</td>
<td>0.79</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Note: This table shows the effect of taxes on steady state hours and welfare in the complete markets/stand-in household model relative to the steady state with $\tau = 0.30$. 

Table 1

Effect of taxes in the stand-in household model.
equilibrium without taxes yields a Pareto efficient allocation. The steady state welfare measure abstracts from transition dynamics, but nonetheless the table shows that steady state welfare is highest when taxes are set to zero. The same studies that produce estimates of the average tax rate on labor equal to 0.30 in the US suggest a value of 0.50 for the continental European economies of Belgium, France, Germany and Italy. 3 Looking at Table 1, this model predicts more than a 20% reduction in hours of work and a welfare cost equal to 12% of steady state consumption when moving from \( \tau = 0.3 \) to \( \tau = 0.5 \).

3. The heterogeneous agents/incomplete markets model

This section describes the heterogeneous agents model that is the focus of this study. The model follows in the tradition of Huggett (1993) and Aiyagari (1994), though is most similar to the one developed in Chang and Kim (2007). It is also very similar to those in Floeden and Linde (2001), and Pijoan-Mas (2006) with the main difference being that we assume indivisible labor. Because coordination problems within organizations often restrict the ability of individuals to work significantly different hours than their coworkers, we believe that the indivisible labor assumption is an appropriate one in contexts that stress idiosyncratic cross-sectional heterogeneity. Pijoan-Mas (2006) concludes that the labor supply elasticity must be very small in order to reconcile the large cross-sectional differences in productivity with the relatively small cross-sectional differences in hours. However, if the low variance in hours is due to technological factors that require some degree of coordination of hours across workers, this procedure will bias the estimated preference parameters. For this reason we think that the indivisible labor assumption is preferable in this context.4

There is a continuum with unit mass of individuals, indexed by \( i \), each with preferences described by

\[
\sum_{t=0}^{\infty} \beta^t [\log(c_{it} - \delta h_{it})]
\]

Because of the assumption of indivisible labor there is no loss in generality in assuming linear disutility from working. There is a Cobb–Douglas aggregate production function that uses capital \((K_t)\) and labor \((L_t)\) to produce output, \(Y_t = K_t^\alpha L_t^{1-\alpha}\), where upper case letters are used to denote aggregate values, since individual and aggregate values will no longer be the same. Individual productivity is stochastic. Productivity for individual \( i \) in period \( t \), denoted by \( e_{it} \), is assumed to follow the process:

\[
\log e_{it+1} = \rho \log e_{it} + \epsilon_{it+1}
\]

where \( \epsilon_{it} \) is normally distributed, with mean and standard deviation given by \( \mu_e \) and \( \sigma_e \) respectively. Denote the density function for \( \epsilon \) by \( f(\epsilon) \). Realizations are iid across individuals. Period \( t \) productivity is realized before any period \( t \) decisions are made. Because of the idiosyncratic productivity shocks, aggregate labor input \( L_t \) is a weighted integral of time inputs:

\[
L_t = \int_0^1 e_{it} h_{it} di
\]

As before, output can be used as either consumption or investment and capital depreciates at rate \( \delta \). As in the previous section, government taxes labor income at constant rate \( \tau \) and uses the proceeds to fund a lump-sum transfer that is uniform across all individuals, subject to a period-by-period balanced budget constraint.

The market structure is as follows. At each date \( t \) there are factor markets for capital and labor services, and a market for output. There are no insurance markets, but individuals can self-insure by accumulating capital. Individual capital holdings are required to be non-negative, which implies that there are effectively no markets for borrowing and lending, i.e., capital accumulation is the only channel through which individuals can move resources across periods. Once again, the analysis focuses on steady state equilibrium. Let \( w \) and \( r \) denote the steady state equilibrium rental prices for labor services and capital services. An individual with idiosyncratic productivity \( e \) who chooses to work will then earn labor income equal to \( ew \). An individual who enters the period with capital holdings \( k \) and has a current productivity realization of \( e \) faces the following one period budget constraint:

\[
c + k' = (1-\delta)k + rk + (1-\tau)ew + T
\]

where \( k' \) is next period’s capital.

The state vector for an individual is \( s=(k,e) \). In steady state, the Bellman equation for an individual is

\[
V(k,e) = \max_{c,k} \left\{ \log c - ah + \beta \int V(k',e') f(e') de' \right\}
\]

s.t. \( c + k' = (1-\delta)k + rk + (1-\tau)ew + T, \quad c \geq 0, \quad k' \geq 0, \quad h \in [0,1] \)

---

3. Mendoza et al. (1994) was an early contribution to this literature. More recently McDaniel (2006) has produced longer time series using a variation on the method used by Mendoza et al. along the lines of what Prescott (2004) did.

4. While the benchmark model in the previous section did not assume indivisible labor for the representative household, note that this is consistent with the work of Chang and Kim (2007) who found that in a model with indivisible labor, idiosyncratic shocks and incomplete markets, the Frisch elasticity for aggregate labor supply is not infinite.
Since the analysis will consider how changes in $\tau$ affect the steady state equilibrium, it will be useful to include $\tau$ as an argument of functions that describe the equilibrium. Specifically, let $c(s; \tau), k(s; \tau),$ and $h(s; \tau)$ denote the individual decision rules in steady state for a given value of $\tau,$ and let $\mu(s; \tau)$ be the measure of individuals across state vectors as a function of $\tau.$ Steady state aggregates are then expressed as

$$H(\tau) = \int h(s; \tau) d\mu,$$

$$L(\tau) = \int c(s; \tau) d\mu,$$

$$K(\tau) = \int k d\mu.$$

Relative to a baseline value of $\tau,$ the steady state change in welfare associated with a change of $\tau$ to some other value $\tau'$ is again defined as the proportional increase in consumption for all agents that would be required to equate the welfare measures, i.e., the value of $\Delta$ that solves:

$$\frac{1}{1-\beta} \int [\log(c(s; \tau')) - zh(s; \tau')] d\mu = \frac{1}{1-\beta} \int [\log((1+\Delta)c(s; \tau')) - zh(s; \tau')] d\mu'$$

where $\mu'$ is the measure of individuals over state vectors for $\tau'.$ As before, a negative value of $\Delta$ indicates a welfare gain and a positive value of $\Delta$ indicates a welfare loss relative to the baseline.

4. Quantitative analysis of taxes and transfers

This section considers the quantitative effects of changes in the tax and transfer system on the steady state equilibrium of the heterogeneous agents model with incomplete markets.

4.1. Calibration

Relative to the benchmark model described in Section 2, there are three additional parameters in the incomplete markets model: the two parameters of the stochastic process ($\mu_s$ and $\sigma_s$) and the fixed workweek length, $\overline{h}.$ Subject to choosing these parameters, the remaining parameters are chosen to match the same aggregate targets as in Section 2. A sizeable literature estimates idiosyncratic shock processes, including Card (1994), Floden and Linde (2001), French (2005), Chang and Kim (2006), Heathcote et al. (2007) and Low et al. (2010). These papers typically estimate this process for prime aged males. The common feature of this literature is that the process is found to be very persistent, with most estimates 0.9 or higher. The benchmark case adopts an intermediate value of 0.94. The variance of the idiosyncratic shock is assumed to be 0.205.5

The value of $\overline{h}$ is chosen to be consistent with mean workweek length for full time male workers. Consistent with the earlier calibration, total time devoted to work is set to 0.33 in the steady state. Because the model is computed assuming a discrete state space, one cannot target an arbitrary desired employment level. In the benchmark calibration, $\overline{h}$ is set to 0.407 and the steady state employment to population ratio is equal to 0.818.6 Assuming a weekly time endowment of 100 h for discretionary time, the value for $h$ is chosen to be consistent with mean workweek length for full time male workers. Consistent with the earlier calibration, total time devoted to work is set to 0.33 in the steady state. Because the model is computed assuming a discrete state space, one cannot target an arbitrary desired employment level. 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The approximation method used to find the steady state equilibrium is standard. The individual decision problem is solved using value function iteration. The value function is approximated by a piece-wise linear function. Golden section search is used on a discrete grid over the state space. Monte Carlo simulation is used to approximate the distribution of the economy, with a search over prices and transfers until the market clearing condition and budget balance conditions are satisfied.7

Note that by virtue of the calibration procedure, the steady state equilibrium in the incomplete markets model looks just like the steady state equilibrium in the representative household model along several dimensions. In particular, each has the same capital to output ratio (2.61), the same real interest rate (4%), the same consumption–output ratio (0.75), and the same fraction of total time devoted to work (0.33). The incomplete markets model also has predictions for some distributional statistics. There are many papers in the literature that have documented the properties of the wealth distribution in a similar model, though the closest is the recent paper by An et al. (2009). While our model differs from theirs in some details (e.g., we use a more persistent shock process, we have a tax and transfer scheme, they have a weaker borrowing constraint), the distributional statistics in our calibrated model look quite similar to theirs. For completeness we report statistics for the steady state wealth and labor earnings distributions across wealth quintiles in Table 2, along with statistics reported in An et al. (2009) based on a sample of households from the PSID. The table indicates that the model does an excellent job in capturing the distribution of both wealth and labor earnings across the quintiles.8

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5 Chang and Kim (2006) estimate a much lower persistence, but this is because they do not allow for measurement error in the form of a purely transitory shock.

6 The average value of the employment to population ratio for males aged 15–64 in the US over the last 50 years is also roughly 0.80.

7 Note that by virtue of the calibration procedure, the steady state equilibrium in the incomplete markets model looks just like the steady state equilibrium in the representative household model along several dimensions. In particular, each has the same capital to output ratio (2.61), the same real interest rate (4%), the same consumption–output ratio (0.75), and the same fraction of total time devoted to work (0.33). The incomplete markets model also has predictions for some distributional statistics. There are many papers in the literature that have documented the properties of the wealth distribution in a similar model, though the closest is the recent paper by An et al. (2009). While our model differs from theirs in some details (e.g., we use a more persistent shock process, we have a tax and transfer scheme, they have a weaker borrowing constraint), the distributional statistics in our calibrated model look quite similar to theirs. For completeness we report statistics for the steady state wealth and labor earnings distributions across wealth quintiles in Table 2, along with statistics reported in An et al. (2009) based on a sample of households from the PSID. The table indicates that the model does an excellent job in capturing the distribution of both wealth and labor earnings across the quintiles.

8 As noted by several previous papers in the literature, this class of models does not do a good job of capturing the concentration of wealth within the top quintile. See, for example, the analysis in Krusell and Smith (1998), who added discount factor shocks in order to better match this feature. From the perspective of understanding aggregate labor supply, we do not feel that accounting for the existence of individuals such as Bill Gates is of primary importance.
4.2. Results

Analogous to the results in Table 1 for the stand-in household model, Table 3 shows how changes in taxes affect steady state hours and welfare relative to the $t = 0.30$ steady state. Aggregate hours of work are again decreasing in the scale of the tax and transfer program. Moreover, comparing the change in hours of work between $t = 0$ and $0.50$ the magnitudes are quite similar for the two models: 46% in the stand-in household model versus 42% in the heterogeneous agent/incomplete markets model. One difference between the predictions of the two models is that the effect of tax increases on aggregate hours is less linear in the model with heterogeneous agents. The incremental effects of increasing taxes are very weakly increasing in Table 1 but are strongly increasing in Table 3. Despite this, from the narrow perspective of assessing the extent to which differences in taxes account for the differences in hours of work between the US and the high-tax economies of continental Europe, both models would support the conclusion that differences in the scale of tax and transfer programs can account for a large share of the differences in aggregate hours of work.

The welfare implications of the two models feature some striking differences. First, in the stand-in household model, increasing $t$ from 0.3 to 0.5 entails a welfare loss of 12%, which is three times larger than the welfare loss in the heterogeneous agents/incomplete markets model. Second, in considering tax decreases relative to the $t = 0.30$ benchmark, not only does the magnitude of the welfare effect change, but also the sign. In particular, welfare is substantially higher in the $t = 0.30$ steady state than in the $t = 0$ steady state, even though hours worked are much lower in the $t = 0.30$ steady state.9

In summary, considering an increase of $t$ from 0.30 to 0.50, the heterogeneous agents/incomplete markets model implies somewhat larger effects on aggregate hours but substantially smaller effects on welfare than the stand-in household model. Considering decreases in taxes from $t = 0.30$ to 0, both models predict increases in hours, but the welfare effects are large and of opposite signs. The remainder of this section looks to shed light on the sources of these differences.

Key to understanding these differences is to appreciate how labor supply interacts with asset accumulation in the heterogeneous agents/incomplete markets model to facilitate consumption smoothing in the face of productivity shocks. It is useful to begin by noting what steady state allocations would look like if there were complete markets for risk sharing. In this case consumption would be constant across individuals and the employment decision rule would take the form of a reservation rule: work if productivity is above some threshold level $e$. While an individual in the incomplete markets model could still adopt this threshold rule, it turns out not to be optimal. Instead, the optimal decision rule for labor supply depends on both assets and productivity and can be described as a reservation rule contingent on assets: for a given level of assets $k$, there is a productivity level $e(k)$ such that the individual works if productivity is above this level. The reservation value is increasing in $k$, reflecting the standard income effect.

Consistent with the discussion in Pijoan-Mas (2006), individuals use both labor supply and asset accumulation as tools to help them smooth consumption in the face of productivity shocks. Specifically, individuals with high realizations of the

Table 2
Earnings and wealth distributions.

<table>
<thead>
<tr>
<th>Wealth quintile</th>
<th>Data Wealth share (%)</th>
<th>Model Wealth share (%)</th>
<th>Earnings share (%)</th>
<th>Data Earnings share (%)</th>
<th>Model Earnings share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>-0.52</td>
<td>0.02</td>
<td>7.51</td>
<td>1.56</td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td>0.50</td>
<td>0.82</td>
<td>11.31</td>
<td>16.68</td>
<td></td>
</tr>
<tr>
<td>Third</td>
<td>5.06</td>
<td>6.7</td>
<td>18.72</td>
<td>20.30</td>
<td></td>
</tr>
<tr>
<td>Fourth</td>
<td>18.74</td>
<td>21.1</td>
<td>24.21</td>
<td>24.21</td>
<td></td>
</tr>
<tr>
<td>Fifth</td>
<td>76.22</td>
<td>71.3</td>
<td>38.23</td>
<td>37.25</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows how quintiles of the wealth and earnings distribution in the calibrated model compare with those calculated for a sample from the PSID.

Table 3
The effect of taxes in the heterogeneous agent economy.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$H(t)/H(0.3)$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>1.15</td>
<td>0.044</td>
</tr>
<tr>
<td>$t = 0.10$</td>
<td>1.12</td>
<td>0.022</td>
</tr>
<tr>
<td>$t = 0.20$</td>
<td>1.08</td>
<td>0.007</td>
</tr>
<tr>
<td>$t = 0.30$</td>
<td>1.00</td>
<td>0.000</td>
</tr>
<tr>
<td>$t = 0.40$</td>
<td>0.89</td>
<td>0.010</td>
</tr>
<tr>
<td>$t = 0.50$</td>
<td>0.73</td>
<td>0.041</td>
</tr>
</tbody>
</table>

Note: This table shows how taxes affect steady state hours and welfare in the heterogeneous agents/incomplete markets economy relative to the steady state with $t = 0.30$.

### 4.2. Results

Analogous to the results in Table 1 for the stand-in household model, Table 3 shows how changes in taxes affect steady state hours and welfare relative to the $t = 0.30$ steady state. Aggregate hours of work are again decreasing in the scale of the tax and transfer program. Moreover, comparing the change in hours of work between $t = 0$ and $0.50$ the magnitudes are quite similar for the two models: 46% in the stand-in household model versus 42% in the heterogeneous agent/incomplete markets model. One difference between the predictions of the two models is that the effect of tax increases on aggregate hours is less linear in the model with heterogeneous agents. The incremental effects of increasing taxes are very weakly increasing in Table 1 but are strongly increasing in Table 3. Despite this, from the narrow perspective of assessing the extent to which differences in taxes account for the differences in hours of work between the US and the high-tax economies of continental Europe, both models would support the conclusion that differences in the scale of tax and transfer programs can account for a large share of the differences in aggregate hours of work.

The welfare implications of the two models feature some striking differences. First, in the stand-in household model, increasing $t$ from 0.3 to 0.5 entails a welfare loss of 12%, which is three times larger than the welfare loss in the heterogeneous agents/incomplete markets model. Second, in considering tax decreases relative to the $t = 0.30$ benchmark, not only does the magnitude of the welfare effect change, but also the sign. In particular, welfare is substantially higher in the $t = 0.30$ steady state than in the $t = 0$ steady state, even though hours worked are much lower in the $t = 0.30$ steady state.9

In summary, considering an increase of $t$ from 0.30 to 0.50, the heterogeneous agents/incomplete markets model implies somewhat larger effects on aggregate hours but substantially smaller effects on welfare than the stand-in household model. Considering decreases in taxes from $t = 0.30$ to 0, both models predict increases in hours, but the welfare effects are large and of opposite signs. The remainder of this section looks to shed light on the sources of these differences.

Key to understanding these differences is to appreciate how labor supply interacts with asset accumulation in the heterogeneous agents/incomplete markets model to facilitate consumption smoothing in the face of productivity shocks. It is useful to begin by noting what steady state allocations would look like if there were complete markets for risk sharing. In this case consumption would be constant across individuals and the employment decision rule would take the form of a reservation rule: work if productivity is above some threshold level $e$. While an individual in the incomplete markets model could still adopt this threshold rule, it turns out not to be optimal. Instead, the optimal decision rule for labor supply depends on both assets and productivity and can be described as a reservation rule contingent on assets: for a given level of assets $k$, there is a productivity level $e(k)$ such that the individual works if productivity is above this level. The reservation value is increasing in $k$, reflecting the standard income effect.

Consistent with the discussion in Pijoan-Mas (2006), individuals use both labor supply and asset accumulation as tools to help them smooth consumption in the face of productivity shocks. Specifically, individuals with high realizations of the
productivity shock tend to work and accumulate assets that will be used later to help smooth consumption in the event of
low productivity shocks. As is well known, this leads to increased accumulation of capital relative to the model with
complete markets. However, if sufficient assets are accumulated, then even a high productivity individual may choose not
to work, leading to inefficiently low employment for high productivity workers. At the other end of the productivity
distribution, if an individual with a low realization of productivity depletes his or her assets sufficiently, then the
individual will work even though productivity is low. Relative to the complete markets outcome, this results in
inefficiently high labor supply among low productivity workers.

Fig. 1 illustrates these labor supply effects by showing the employment rate as a function of productivity in the steady
states corresponding to $\tau = 0$, 0.30, and 0.50 as well as in the complete markets solution. It is instructive to begin by
focusing on the profiles for $\tau = 0$ and complete markets. Two properties are notable. In terms of the distortions to labor
supply noted above, the tendency for inefficiently high labor supply at low levels of productivity is very strong, whereas
the tendency for inefficiently low labor supply at higher levels of productivity is relatively weak. The key point is that the
use of labor supply to smooth consumption in the face of low productivity shocks is quite substantial.10 As a consequence,
the overall level of employment is very different between the $\tau = 0$ steady state and the complete markets steady state: in
the complete markets equilibrium the employment rate is 0.77, whereas in the incomplete markets economy with $\tau = 0$ it
is 0.94, implying that hours of work are more than 20% higher in the incomplete markets economy. It is also of interest to
assess the extent of precautionary savings. One way to gauge this is to compute the ratio of capital to labor services. In the
complete markets economy, the ratio of capital to labor services is 4.05 whereas in the $\tau = 0$ incomplete markets steady
state it is 4.78, implying that capital is almost 20% “too high”. Another way to gauge the extent of precautionary savings is
to compare interest rates. In the $\tau = 0$ incomplete markets economy the interest rate is 3.64%, whereas in the complete
markets economy it is 5.15%.

Now consider what happens as the size of the tax and transfer program is increased. The direct effect of the tax and
transfer system holding employment decisions constant is to (on average) move income from high productivity states to
low productivity states. Intuitively, this is a partial substitute for missing insurance markets and should serve to help
individuals smooth consumption over time. In fact, this effect is very evident. Table 4 shows the standard deviation of log
consumption in the steady state, which also reflects the variation of consumption over time for a given individual.11 The
standard deviation is decreasing in the size of the tax and transfer system, though note that the marginal impact decreases
as the scale of the tax and transfer program increases.

In addition to the direct effect on consumption volatility due to moving income across states, the transfer payment also
interacts with the labor supply and asset accumulation choices that individuals make. Specifically, there is less need to
accumulate assets in high productivity states, and less need to work in low productivity states. Both of these effects work

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10 It is important to note that the extent to which this is true depends very much on the persistence of the shocks. The working paper version of this
paper showed that if the persistence parameter were reduced from 0.94 to 0.50 then the amount of excessive labor supply at lower levels of productivity
is very small, in which case the model behaves much more like the stand-in household model.

11 The standard deviation of log consumption when $\tau = 0$ is only a bit smaller than the standard deviation of simulated log productivity, which is
0.62. (The latter is a bit larger than the theoretical value due to using the Tauchen method to construct the finite grid approximation.) While it may seem
at first that precautionary saving is leading to very little consumption smoothing, it is important to note that the standard deviation of productivity is not
an appropriate benchmark when workers do not work in each period. For example, in the social planner’s solution, workers below a given productivity
threshold will not work and earnings will have a much larger standard deviation than the underlying productivity process.
to lower the reservation values $\sigma(k)$. Fig. 2 shows the work/no work regions of the individual state space for $\tau = 0, 0.30, 0.50$.

Consistent with the previous discussion, an increase in $\tau$ leads to a leftward shift of the curve. Returning to Fig. 1, note that as $\tau$ increases the employment rate profile shifts down. In addition to the consumption smoothing benefits noted above, a downward shift of the employment profile has both a positive and a negative welfare effect relative to the complete markets employment profile. At low levels of productivity, the decrease in employment is a positive change, whereas at higher levels of productivity the decrease is a negative change. This is in sharp contrast to the situation in the stand-in household model. In that model, the decreased incentives for work associated with larger tax and transfer systems represents only a negative force, whereas in the heterogeneous agents/incomplete markets model, the disincentive effects for work among individuals with low productivity realizations represents a positive force. The welfare maximizing level of $\tau$ reflects a balancing of these two effects. For very low values of $\tau$ the key issue is inefficiently high labor supply among low productivity workers, whereas at very high levels of $\tau$ the issue is inefficiently low levels of labor supply for high productivity workers. At the optimal level of $\tau, \tau = 0.30$, each of these distortions is present, though to a lesser degree than for the two extreme tax rates.

Fig. 1 can also help us understand the nonlinear effects of changes in taxes on changes in aggregate hours of work. The figure shows that at any given tax rate, increases in taxes have the largest effect on those individuals who are employed at low productivity realizations. As the scale of the tax and transfer program is increased, the marginal employed individuals are higher in the productivity distribution. For example, when $\tau = 0$, we see that employment rates are high even at the lowest productivity level, so the marginal workers are those at the bottom of the distribution. In contrast, when $\tau = 0.30$, there is no employment at the bottom part of the productivity distribution, so the marginal employed individuals have higher productivity. Because the stationary distribution of productivity is normally distributed, the mass of marginal individuals increases as the marginal group moves from the bottom of the distribution toward the middle of the distribution. As the size of the marginal group varies, so do the aggregate effects.

It is also of interest to explore what is happening to capital accumulation as the tax and transfer system is expanded. Table 5 shows the effects on the capital to income ratio, the capital to labor services ratio and the interest rate. In the stand-in household model the tax and transfer system had no impact on any of these variables. As noted above, because the tax

---

**Table 4**

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\tau = 0$</th>
<th>$\tau = 0.10$</th>
<th>$\tau = 0.20$</th>
<th>$\tau = 0.30$</th>
<th>$\tau = 0.40$</th>
<th>$\tau = 0.50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>std(log $(c)$)</td>
<td>0.59</td>
<td>0.52</td>
<td>0.47</td>
<td>0.46</td>
<td>0.45</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Note: This table shows how the standard deviation of the log steady state consumption distribution is affected by taxes.

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Fig. 2. Decision rules and taxes. Note: This figure plots the steady state decision rules for employment for three different levels of taxes as a function of the two individual state variables, wealth and productivity.

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12 Ljungqvist and Sargent (2008) also emphasize how taxes and incomplete markets interact to influence whose labor supply is affected by changes in taxes. While the general points are similar, the different shock structures and demographics influence the details.
and transfer program offers an alternative to self insurance, one would expect that a tax and transfer system in the incomplete markets model would affect the extent of precautionary savings and hence impact all three variables. Consistent with this, the table reveals that as \( t \) increases, both \( \frac{K}{L} \) and \( \frac{K}{Y} \) decrease while \( r \) increases. However, the size of these effects is relatively small, especially as we increase taxes beyond \( t = 0.3 \).

It is also of interest to ask how taxes affect the distribution of assets. Table 6 provides some information on this. Overall the effects on the distribution are not that large. The main effect is that as taxes increase there is somewhat less wealth concentration in the top quintile.

The main objective in this analysis has been to assess how the choice of model influences conclusions regarding the effects of tax and transfer programs of the magnitude found in several continental European countries on allocations and welfare relative to the US scenario. We believe that the above analysis points to two key findings. First, the effect on hours of work of increases in tax and transfer programs above the US level is broadly similar in the two different models. Second, the welfare implications are dramatically different. In the stand-in household model, the larger tax and transfer system imposes a very large welfare burden, equivalent to 12% of consumption. While the incomplete market/heterogeneous agent economy also implies a welfare loss associated with the larger tax and transfer system, the welfare loss is only one-third as large. An important implication of this model is that tax and transfer programs can be welfare improving. The scale of the US tax and transfer system represents a significant improvement relative to the case of no tax and transfer system, whereas the European scale system is roughly equivalent in terms of welfare to the case of no tax and transfer system. In view of this finding, it is perhaps much less of a challenge to account for the different scales of the tax and transfer programs found in the US and continental Europe.

5. Taxes and productivity

While the main objective of this analysis was to examine how the incomplete markets model would influence predictions regarding the effect of labor tax increases on hours worked and welfare, this section points out one additional implication of interest that concerns cross-country differences in productivity. It is somewhat of a stylized fact that although hours of work are much less in continental Europe than in the US, labor productivity in these countries is very nearly equal (and in some cases even higher) than in the US. Somewhat less publicized is the fact that in several countries, including Australia and Canada, hours of work are very similar to US levels, whereas the level of labor productivity is roughly 15% lower. One could argue that these productivity comparisons seem puzzling. Although theories have not yet been developed that can successfully account for the differences in productivity levels observed across countries, there is some consensus that several factors might serve as barriers to technology adoption, or efficient operation of technologies. Examples include various forms of regulation, the role of government, the presence of unions, etc. For the most part, these factors are more prevalent in the economies of Europe than they are in the US. Countries such as Australia and Canada would seem to be intermediate cases. Viewed from this perspective it is perhaps puzzling that productivity in many European countries is so close to the US levels and higher than in countries such as Canada and Australia.

Table 5
Taxes and capital accumulation.

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( \tau = 0 )</th>
<th>( \tau = 0.10 )</th>
<th>( \tau = 0.20 )</th>
<th>( \tau = 0.30 )</th>
<th>( \tau = 0.40 )</th>
<th>( \tau = 0.50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{K(\tau)}{Y(\tau)} )</td>
<td>1.031</td>
<td>1.018</td>
<td>1.007</td>
<td>1.000</td>
<td>0.990</td>
<td>0.986</td>
</tr>
<tr>
<td>( \frac{K(\tau)}{L(\tau)} )</td>
<td>1.049</td>
<td>1.027</td>
<td>1.012</td>
<td>1.000</td>
<td>0.985</td>
<td>0.978</td>
</tr>
<tr>
<td>( \frac{r(\tau)}{r(0.3)} )</td>
<td>0.0041</td>
<td>-0.0024</td>
<td>-0.0011</td>
<td>0</td>
<td>0.0008</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

Note: This table shows how capital accumulation and interest rates are affected by taxes in the heterogeneous agents/incomplete markets model relative to the steady state with \( \tau = 0.3 \).

Table 6
Taxes and the wealth distributions.

<table>
<thead>
<tr>
<th>Wealth quintile</th>
<th>Wealth share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau = 0 )</td>
<td>( \tau = 0.30 )</td>
</tr>
<tr>
<td>First</td>
<td>0.02</td>
</tr>
<tr>
<td>Second</td>
<td>1.35</td>
</tr>
<tr>
<td>Third</td>
<td>6.79</td>
</tr>
<tr>
<td>Fourth</td>
<td>19.88</td>
</tr>
<tr>
<td>Fifth</td>
<td>71.97</td>
</tr>
</tbody>
</table>

Note: This table shows how the wealth distribution in the heterogeneous agents/incomplete markets model is affected by taxes.
The table shows values of hours worked per adult and output per hour relative to the US for the period 2003–2007. Data are from the OECD and GGDC. Note: This table shows how taxes affect steady state output per hour in the heterogeneous agents/incomplete markets model, relative to the steady state with \( t = 0.3 \).

Table 7
Productivity and hours relative to the US.

<table>
<thead>
<tr>
<th>Hours relative to US</th>
<th>( \leq 0.85 )</th>
<th>&gt; 0.85</th>
<th>Y/H relative to US</th>
<th>( \leq 0.85 )</th>
<th>&gt; 0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ita(0.65)</td>
<td>Swe(0.86)</td>
<td></td>
<td>NZ(0.61)</td>
<td>Den(0.90)</td>
<td></td>
</tr>
<tr>
<td>Fra(0.67)</td>
<td>UK(0.87)</td>
<td></td>
<td>Jap(0.71)</td>
<td>Net(0.90)</td>
<td></td>
</tr>
<tr>
<td>Bel(0.69)</td>
<td>Aus(0.92)</td>
<td></td>
<td>Swi(0.79)</td>
<td>Aus(0.82)</td>
<td></td>
</tr>
<tr>
<td>Net(0.72)</td>
<td>NZ(0.94)</td>
<td></td>
<td>Can(0.82)</td>
<td>Ito(0.91)</td>
<td></td>
</tr>
<tr>
<td>Aut(0.78)</td>
<td>Can(0.94)</td>
<td></td>
<td>Bel(1.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Den(0.84)</td>
<td>Swe(0.96)</td>
<td></td>
<td>Jap(0.99)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Swi(0.83)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>UK(0.85)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows values of hours worked per adult and output per hour relative to the US for the period 2003–2007. Data are from the OECD and GGDC.

Table 8
Taxes and labor productivity (values are relative to \( t = 0.3 \)).

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( H )</th>
<th>( L )</th>
<th>( Y )</th>
<th>( Y/H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.15</td>
<td>1.08</td>
<td>1.10</td>
<td>0.96</td>
</tr>
<tr>
<td>0.10</td>
<td>1.12</td>
<td>1.06</td>
<td>1.07</td>
<td>0.96</td>
</tr>
<tr>
<td>0.20</td>
<td>1.08</td>
<td>1.04</td>
<td>1.04</td>
<td>0.97</td>
</tr>
<tr>
<td>0.30</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.40</td>
<td>0.89</td>
<td>1.00</td>
<td>1.00</td>
<td>1.05</td>
</tr>
<tr>
<td>0.50</td>
<td>0.73</td>
<td>0.94</td>
<td>0.94</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Note: This table shows how taxes affect steady state output per hour in the heterogeneous agents/incomplete markets model, relative to the steady state with \( t = 0.3 \).

The heterogeneous agents/incomplete markets model offers a perspective on these cross country comparisons that challenges the conventional wisdom regarding the productivity catch up of continental Europe.

To explore this in more detail we examine the relationship between output per hour and total hours worked per person of working age in a recent cross section. For each series the average over the five year period 2003–2007 is computed so as to eliminate the effect of year to year fluctuations, as well as avoid the effects of the 2001 recession and the recent downturn. The analysis focuses on high productivity countries, and adopts a lower threshold of 0.60 of the US level as the cutoff. This leaves a sample with 14 countries. For each country, values for each statistic are expressed relative to the US. Table 7 presents the data for relative hours of work and relative output per hour. In each case the countries are separated into two bins, those with values less than 0.85 and those with values greater than 0.85.

Looking at the table it is striking that the relative positions for the two values are basically flipped. In fact, the correlation of the two measures is −0.83. A simple OLS regression of relative output per hour on relative hours and a constant gives a coefficient of −0.84 on relative hours with a standard error of 0.16 and an \( R^2 \) of 0.68. The regression equation “predicts” that average productivity in these countries would only be about 80% of the US level if hours worked were the same as in the US.

It is of course possible that the above statistical relationship does not reflect any underlying economic connection. But the earlier analysis does offer one explanation for why this relationship does reflect a common underlying economic mechanism. To see why, recall that in the incomplete markets/heterogeneous agents model, the employment effects of increased taxes are strongest at the lower end of the productivity distribution. This implies that expansions of the tax and transfer system lead to selection effects, thereby creating discrepancy between hours and labor services. Table 8 shows that these effects are large.

As the table shows, increasing taxes from \( \tau = 0.30 \) to 0.50 leads to an apparent 15% increase in output per hour. Countries such as Canada and Australia have hours worked similar to the US and between 25% and 30% higher than countries like Belgium, France and Italy. At the same time, output per hour in Australia and Canada is roughly 15% lower than in these other countries. If differences in hours of work are due to differences in the scale of tax and transfer systems, differences in the level and composition of tax and transfer systems is much larger than the selection effect.

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13 GDP per worker is from the GGDC. Output per hour is then computed using data on annual hours worked per worker in employment, also from the GGDC. Total hours worked are from the GGDC and the size of the population aged 15–64 is from the OECD.

14 Germany is excluded from the analysis for the reason that German productivity decreased quite significantly following reunification. Prior to reunification Germany also had productivity levels similar to the US. Relative to the patterns found for the other countries, pre-unification Germany matches up very well, whereas post-unification Germany is somewhat of an outlier.

15 Given our assumption of indivisible labor it may seem more appropriate to focus on employment rate differences rather than hours. It turns out that the relationship between employment and productivity is similar.

16 Note that these effects are much larger than those suggested by Blanchard (2004).

17 Recall that there is also a small decrease in \( K/L \) as \( \tau \) is increased from 0.30 to 0.50, which partially offsets the selection effect.
then the model suggests that countries such as Belgium, France and Italy should have productivity levels about 15% higher than those in Australia and Canada, which is what is found in the data. In other words, the model supports a view that says that most of these countries have “true” productivity levels that are lower than the US, but that the differences between the US and several European countries is masked by the selection effects associated with more generous transfer systems.\(^{18}\)

6. Conclusion

This paper has examined the effects of tax and transfer programs in a model characterized by idiosyncratic shocks and incomplete markets, with a particular focus on assessing the extent to which the effects on allocations and welfare differ from those present in the stand-in household model. The effect of a 20 point increase in the tax rate on hours of work is fairly similar in the two settings. However, the welfare effects are dramatically different. In the incomplete markets model a positive tax and transfer scheme is optimal, whereas in the stand-in household model the optimal size of the tax and transfer scheme is zero. Moreover, the welfare loss associated with having taxes set too high is much smaller in the incomplete markets model. It is also noteworthy that the incomplete markets model predicts large effects of taxes on average labor productivity, because of the important selection effects present in the model. These effects may imply that countries in continental Europe still face a substantial productivity gap with the US in terms of TFP. We conclude from this exercise idiosyncratic shocks and incomplete markets are important features to be considered in the context of assessing tax and transfer programs.

Acknowledgments

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at 10.1016/j.jmoneco.2010.08.008.

References


\(^{18}\) In this exercise we assume that countries are identical except for the tax and transfer scheme. It is possible that the idiosyncratic shock process also differs across countries. Floden and Linde (2001) focused on differences in this process between Sweden and the US. An important issue to keep in mind is that if the selection effects are large then great care must be exercised in estimating differences in shock processes across countries.